

Parallel Product of Maps and the Construction of Regular Maps

In my time as a PhD research student at The Open University, I delved into the fascinating concepts of parallel products of maps and the construction of regular maps. The use of parallel products served as a pivotal tool in understanding the interplay between various mathematical structures, particularly in the realm of topology, algebra and graph theory. By examining the properties of these maps, I aimed to uncover new insights into their applications and potential for simplifying complex systems.

My Thesis considered construction of regular and orientably-regular maps with specified external symmetries, in particular, with specified invariance to rotational powers. Given a map M consisting of vertices of degree k and faces of length m and an integer j relatively prime to k , the operator of a j th rotational power constructs a new map $M(j)$ from M by replacing all the local rotations by their j th powers. If M is (orientably-) regular, then so is $M(j)$. If $M(j)$ is isomorphic to M , then j is an exponent of M . The collection of exponents of M forms a group isomorphic to a subgroup of the group of units mod k .

The two key questions considered in my research involved the existence of an (orientably-) regular map of valency k with exponent group U , given K and a group U of units mod k as well as the existence of an (orientably-) regular map of type (k,m) with a trivial exponent group.

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